The dipole form of the BFKL kernel in supersymmetric Yang–Mills theories *

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Abstract

The dipole (Möbius) representation of the colour singlet BFKL kernel in the next-to-leading order is found in supersymmetric Yang–Mills theories. Ambiguities of this form and its conformal properties are discussed.

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* *Work supported in part by the RFBR grant 07-02-00953, in part by the RFBR–MSTI grant 06-02-72041, in part by INTAS and in part by Ministero Italiano dell'Università e della Ricerca.*

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1 Introduction

For scattering of calourless objects the kernel $\hat{\mathcal{K}}$ of the BFKL [1] equation can be taken in a special form, which we call dipole form [2]. This form is obtained by transformation of the kernel $\langle\vec{q}_1\vec{q}_2|\hat{\mathcal{K}}|\vec{q}_1^{\,\prime}\vec{q}_2^{\,\prime}\rangle$ from the space of transverse momenta $\vec{q}_i$, $\ i=1,2$, where it is originally defined, to the space of transverse coordinates $\vec{r}_i$, with subsequent rejection of the terms proportional to $\delta(\vec{r}_1^{\,\prime}-\vec{r}_2^{\,\prime})$ in $\langle\vec{r}_1\vec{r}_2|\hat{\mathcal{K}}|\vec{r}_1^{\,\prime}\vec{r}_2^{\,\prime}\rangle$ and revising terms not depending either on $\vec{r}_1$ or on $\vec{r}_2$ so as to make the kernel vanishing at $\vec{r}_1=\vec{r}_2$ (see Ref. [2] for details). In another words, the kernel can be considered as acting in the space of functions, vanishing at $\vec{r}_1=\vec{r}_2$. This space is called Möbius space [3] and the dipole form is called also Möbius form. We will use both names and denote this form as $\langle\vec{r}_1\vec{r}_2|\hat{\mathcal{K}}_d|\vec{r}_1^{\,\prime}\vec{r}_2^{\,\prime}\rangle$.

The Möbius form is interesting for several reasons. One of them is the investigation of the inter-relation of the BFKL approach and the colour dipole model [4], where the nonlinear generalization of the BFKL equation [5] is obtained. A clear undestanding of this inter-relation can be helpful for the further development of the theoretical description of small-x processes. Another reason is the study of conformal properties of the kernel. It is known [6] that in the leading order (LO) the BFKL kernel in the Möbius space is conformal invariant, property which is extremely useful for finding solutions of the BFKL equation. Evidently, in the next-to-leading order (NLO) in QCD conformal invariance is violated by renormalization. However, one can expect conformal invariance of the NLO BFKL kernel in supersymmetric extensions of QCD. An additional reason is a possible simplification of the kernel in the Möbius form compared to the momentum space representation.

In Refs. [2], [7] and [8] the Möbius form of the quark and gluon parts of the kernel correspondingly was found for the QCD case. It was shown that the dipole form of the quark part agrees with the result obtained in Ref. [9] by the direct calculation of the corresponding contribution to the Balitsky-Kovchegov (BK) kernel [5]. It was shown also that the Möbius form of the "Abelian" piece of the quark part, which is not associated with renormalization and has the most complicated form in the momentum space, is strongly simplified and is conformal invariant. As well as the quark part, the gluon part in the dipole form is greatly simplified compared with the momentum space representation, although conformal invariance of this part is broken by several terms, among which those not associated with renormalization. However, the ambiguity of the NLO kernel [2] (analogous to the ambiguity of the NLO anomalous dimensions), allowing the transformations

$$\hat{\mathcal{K}} \to \hat{\mathcal{K}} - [\hat{\mathcal{K}}^B\hat{O}] \ , \tag{1}$$

where $\hat{\mathcal{K}}^B$ is the LO kernel and $\hat{O}\sim g^2$, leaves a hope for conformal invariance of the piece not related to renormalization.

The aim of this work is to consider supersymmetric Yang–Mills theories. We analyze the

generalizations of the BFKL kernel for these theories and find their dipole forms.

2 An overall view of the Möbius form

We use the same notation as in Ref. [2]: $\vec{q}_i^{\,\prime}$ and $\vec{q}_i$, $i=1,2$, represent the transverse momenta of Reggeons in initial and final t-channel states, while $\vec{r}_i^{\,\prime}$ and $\vec{r}_i$ are the corresponding conjugate coordinates. The state normalization is

$$\langle\vec{q}|\vec{q}^{\,\prime}\rangle = \delta(\vec{q}-\vec{q}^{\,\prime}) \ , \qquad \langle\vec{r}|\vec{r}^{\,\prime}\rangle = \delta(\vec{r}-\vec{r}^{\,\prime}) \ , \tag{2}$$

so that

$$\langle\vec{r}|\vec{q}\rangle = \frac{e^{i\vec{q}\vec{r}}}{(2\pi)^{1+\epsilon}} \ , \tag{3}$$

where $\epsilon = (D-4)/2$; $D-2$ is the dimension of the transverse space and it is taken different from 2 for the regularization of divergences. We shall also use the notation $\vec{q} = \vec{q}_1+\vec{q}_2$, $\vec{q}^{\,\prime} = \vec{q}_1^{\,\prime}+\vec{q}_2^{\,\prime}$; $\vec{k} = \vec{q}_1-\vec{q}_1^{\,\prime} = \vec{q}_2^{\,\prime}-\vec{q}_2$.

In the LO, discussed in detail in Ref. [2], the dipole form is written as

$$\langle\vec{r}_1\vec{r}_2|\hat{\mathcal{K}}_d^{LO}|\vec{r}_1^{\,\prime}\vec{r}_2^{\,\prime}\rangle = \frac{\alpha_s(\mu)N_c}{2\pi^2}\int d\vec{\rho}\,\frac{\vec{r}_{12}^{\;2}}{\vec{r}_{1\rho}^{\;2}\vec{r}_{2\rho}^{\;2}}\left[\delta(\vec{r}_{11'})\delta(\vec{r}_{2'\rho})+\delta(\vec{r}_{1'\rho})\delta(\vec{r}_{22'})-\delta(\vec{r}_{11'})\delta(r_{22'})\right] . \tag{4}$$

Here and below $\vec{r}_{ij} = \vec{r}_i-\vec{r}_j$, $\vec{r}_{i'j'} = \vec{r}_i^{\,\prime}-\vec{r}_j^{\,\prime}$, $\vec{r}_{ij'} = \vec{r}_i-\vec{r}_j^{\,\prime}$, $\vec{r}_{i\rho} = \vec{r}_i-\vec{\rho}$. Note that the integrand in Eq. (4) contains ultraviolet singularities at $\vec{\rho}=\vec{r}_1$ and $\vec{\rho}=\vec{r}_2$ which cancel in the sum of the contributions assuming that the kernel acts in the Möbius space. The coefficient of $\delta(\vec{r}_{11'})\delta(\vec{r}_{22'})$ is written in the integral form in order to make the cancellation evident. The singularities do not permit us to perform the integration in this coefficient. In Eq. (4) $\alpha_s(\mu) = g_\mu^2/(4\pi)$, where g_μ is the renormalized coupling and the argument μ is shown because we want to present the NLO corrections, which depend on this argument.

A general view of the NLO dipole form is

$$\langle\vec{r}_1\vec{r}_2|\hat{\mathcal{K}}_d^{NLO}|\vec{r}_1^{\,\prime}\vec{r}_2^{\,\prime}\rangle = \frac{\alpha_s^2(\mu)N_c^2}{4\pi^3}\left[\delta(\vec{r}_{11'})\delta(\vec{r}_{22'})\int d\vec{\rho}\, g^0(\vec{r}_1,\vec{r}_2;\vec{\rho})\right.$$
$$\left.+\,\delta(\vec{r}_{11'})g(\vec{r}_1,\vec{r}_2;\vec{r}_2^{\,\prime})+\delta(\vec{r}_{22'})g(\vec{r}_2,\vec{r}_1;\vec{r}_1^{\,\prime})+\frac{1}{\pi}g(\vec{r}_1,\vec{r}_2;\vec{r}_1^{\,\prime},\vec{r}_2^{\,\prime})\right] \ , \tag{5}$$

with the functions g turning into zero when their first two arguments coincide. As well as in the LO, the coefficient of $\delta(\vec{r}_{11'})\delta(\vec{r}_{22'})$ is written in the integral form in order to make evident the

cancellation of the ultraviolet divergencies. Of course, the integrand in this form is not unique. We shall use the equalities

$$\int \frac{d\vec{\rho}}{\vec{r}^{\,2}_{1\rho}\vec{r}^{\,2}_{2\rho}} \left[\vec{r}^{\,2}_{12} \ln\left(\frac{\vec{r}^{\,2}_{1\rho}\vec{r}^{\,2}_{2\rho}}{\vec{r}^{\,4}_{12}}\right) + \left(\vec{r}^{\,2}_{1\rho} - \vec{r}^{\,2}_{2\rho}\right) \ln\left(\frac{\vec{r}^{\,2}_{1\rho}}{\vec{r}^{\,2}_{2\rho}}\right)\right] = 0 \,, \tag{6}$$

$$\frac{1}{4\pi}\int d\vec{\rho} \frac{\vec{r}^{\,2}_{12}}{\vec{r}^{\,2}_{1\rho}\vec{r}^{\,2}_{2\rho}} \ln\left(\frac{\vec{r}^{\,2}_{1\rho}}{\vec{r}^{\,2}_{12}}\right) \ln\left(\frac{\vec{r}^{\,2}_{2\rho}}{\vec{r}^{\,2}_{12}}\right) = \frac{1}{4\pi}\int d\vec{\rho} \frac{1}{\vec{r}^{\,2}_{2\rho}} \ln\left(\frac{\vec{r}^{\,2}_{1\rho}}{\vec{r}^{\,2}_{2\rho}}\right) \ln\left(\frac{\vec{r}^{\,2}_{1\rho}}{\vec{r}^{\,2}_{12}}\right) \,, \tag{7}$$

for modifying this integrand.

It is known (see Ref. [10] and references therein) that in four dimensions only three types of sypersymmetric Yang-Mills theories ($N = 1$, $N = 2$ and $N = 4$ theories) are possible. The simplest, $N = 1$ theory, contains the Yang-Mills fields (gluons) and the Maiorana spinors (gluinos). At $N = 2$ and $N = 4$ besides the vectors and spinors there are also scalars and pseudoscalars. For our purposes there is no difference between scalars and pseudoscalars. In the following we shall call both of them scalars. All particles are contained in the adjoint representation of the colour group. The number of gluinos n_M is equal to N, the number of scalars n_S is equal to $2(N-1)$. We shall use the relation between the bare coupling g and the renormalized one g_μ in the $\overline{\text{MS}}$ form

$$g = g_\mu \mu^{-\epsilon}\left[1 + \frac{g_\mu^2 N_c}{(4\pi)^2}\left(\frac{11}{3} - \frac{2}{3}n_M - \frac{1}{6}n_S\right)\frac{1}{2\epsilon}\right] \,, \tag{8}$$

where $n_M = N, \quad n_S = 2(N-1)$. It is known that the standard form of the dimensional regularization violates supersymmetry, because it violates the equality of Bose and Fermi degrees of freedom. The modification of the dimensional regularization which preserves sypersymmetry (it is called dimensional reduction) was suggested in Ref. [11]. Unfortunately, it contains the inherent inconsistency [12]. However, the inconsistency becomes apparent only at the three-loop level and can be removed with the violation of supersymmetry in the highest orders [13]. Therefore the dimensional reduction is widely used for practical calculations. With our accuracy the dimensional reduction is equivalent to the dimensional regularization with the number of scalars n_S equal to $2(N-1)-2\epsilon$. It follows from Eq. (8) that the use of the dimensional reduction instead of the dimensional regularization is equivalent to the finite charge remornalizaion

$$\alpha_s(\mu) \to \alpha_s(\mu)\left(1 - \frac{\alpha_s(\mu)N_c}{12\pi}\right). \tag{9}$$

In Refs. [2] and [8] the operator transformation (1), with the operator $\hat{O}$ associated with the charge renormalization, was applied to the kernel in the momentum space defined by the

prescriptions given in Ref. [14]. It occurs [2] that this transformation simplifies considerably the quark part. We shall use the SUSY generalization of this transformation, with

$$\hat{O} = \hat{O}_G + \hat{O}_M + \hat{O}_S = -\frac{\alpha_s N_c}{8\pi}\left(\frac{11}{3} - \frac{2}{3}n_M - \frac{1}{6}n_S\right)\ln\left(\frac{\hat{\vec{q}}_1^{\,2}\hat{\vec{q}}_2^{\,2}}{\mu^4}\right). \tag{10}$$

At the $N = 4$ supersymmetry $11/3 - (2/3)n_M - (1/6)n_S = 0$, α_s does not depend on μ and $\hat{O} = 0$.

3 Gluon and gluino parts

The gluon contribution to the kernel is the same as in QCD. We denote this part by the subscript G. Using Eq. (7), from the results of Ref. [8] we obtain

$$g_G^0(\vec{r}_1, \vec{r}_2; \vec{\rho}) \;= -g_G(\vec{r}_1, \vec{r}_2; \vec{\rho}) \;+ \frac{1}{2}\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{1\rho}^{\,2}\vec{r}_{2\rho}^{\,2}}\ln\left(\frac{\vec{r}_{1\rho}^{\,2}}{\vec{r}_{12}^{\,2}}\right)\ln\left(\frac{\vec{r}_{2\rho}^{\,2}}{\vec{r}_{12}^{\,2}}\right)\;, \tag{11}$$

$$\begin{aligned} g_G(\vec{r}_1, \vec{r}_2; \vec{\rho}) \;&= \frac{11}{6}\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{2\rho}^{\,2}\vec{r}_{1\rho}^{\,2}}\ln\left(\frac{\vec{r}_{12}^{\,2}}{r_G^2}\right) + \frac{11}{6}\frac{\vec{r}_{1\rho}^{\;2} - \vec{r}_{2\rho}^{\;2}}{\vec{r}_{1\rho}^{\;2}\vec{r}_{2\rho}^{\;2}}\ln\left(\frac{\vec{r}_{2\rho}^{\;2}}{\vec{r}_{1\rho}^{\;2}}\right) \\ &+ \frac{1}{2\vec{r}_{2\rho}^{\,2}}\ln\left(\frac{\vec{r}_{1\rho}^{\,2}}{\vec{r}_{2\rho}^{\,2}}\right)\ln\left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{1\rho}^{\,2}}\right) - \frac{\vec{r}_{12}^{\,2}}{2\,\vec{r}_{2\rho}^{\,2}\vec{r}_{1\rho}^{\,2}}\ln\left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{2\rho}^{\,2}}\right)\ln\left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{1\rho}^{\,2}}\right), \end{aligned} \tag{12}$$

where

$$\ln r_G^2 = 2\psi(1) - \ln\frac{\mu^2}{4} - \frac{3}{11}\left(\frac{67}{9} - 2\zeta(2)\right). \tag{13}$$

Both $g_G^0(\vec{r}_1, \vec{r}_2; \vec{\rho})$ and $g_G(\vec{r}_1, \vec{r}_2; \vec{\rho})$ vanish at $\vec{r}_1 = \vec{r}_2$. Then, these functions turn into zero for $\vec{\rho}^{\,2} \to \infty$ faster than $(\vec{\rho}^{\,2})^{-1}$ to provide the infrared safety. The ultraviolet singularities of these functions at $\vec{\rho} = \vec{r}_2$ and $\vec{\rho} = \vec{r}_1$ cancel assuming that the kernel acts in the Möbius space.

The most complicated contribution is $g_G(\vec{r}_1, \vec{r}_2; \vec{r}_1^{\,\prime}, \vec{r}_2^{\,\prime})$. It can be written as

$$\begin{aligned} g_G(\vec{r}_1, \vec{r}_2; \vec{r}_1^{\,\prime}, \vec{r}_2^{\,\prime}) = &\frac{1}{\vec{r}_{1'2'}^{\,4}}\left(\frac{\vec{r}_{11'}^{\,2}\,\vec{r}_{22'}^{\,2}}{d}\ln\left(\frac{\vec{r}_{12'}^{\,2}\,\vec{r}_{21'}^{\,2}}{\vec{r}_{11'}^{\,2}\vec{r}_{22'}^{\,2}}\right) - 1\right) + \frac{\vec{r}_{12}^{\;2}}{2d\,\vec{r}_{1'2'}^{\;2}}\ln\left(\frac{\vec{r}_{12'}^{\;2}\vec{r}_{21'}^{\;2}}{\vec{r}_{11'}^{\;2}\;\vec{r}_{22'}^{\;2}}\right)\left(\frac{\vec{r}_{12}^{\;2}\vec{r}_{1'2'}^{\;2}}{\vec{r}_{11'}^{\;2}\;\vec{r}_{22'}^{\;2}} - 4\right) \\ &+\frac{1}{2\vec{r}_{12'}^{\;2}\vec{r}_{21'}^{\;2}\;\vec{r}_{1'2'}^{\;2}}\left(2(\vec{r}_{12'}\vec{r}_{21'})\ln\left(\frac{\vec{r}_{11'}^{\;2}\vec{r}_{22'}^{\;2}}{\vec{r}_{12}^{\;2}\;\vec{r}_{1'2'}^{\;2}}\right) + \vec{r}_{12'}^{\;2}\ln\left(\frac{\vec{r}_{12}^{\;2}\vec{r}_{22'}^{\;2}}{\vec{r}_{1'2'}^{\;2}\vec{r}_{11'}^{\;2}}\right) + \vec{r}_{21'}^{\;2}\ln\left(\frac{\vec{r}_{12}^{\;2}\vec{r}_{11'}^{\;2}}{\vec{r}_{1'2'}^{\;2}\vec{r}_{22'}^{\;2}}\right)\right) \\ &+\frac{1}{2\vec{r}_{11'}^{\;2}\vec{r}_{22'}^{\;2}\;\vec{r}_{1'2'}^{\;2}}\left((\vec{r}_{11'}^{\;2} + \vec{r}_{22'}^{\;2} + \vec{r}_{12}^{\;2})\ln\left(\frac{\vec{r}_{12}^{\;2}\;\vec{r}_{1'2'}^{\;2}}{\vec{r}_{12'}^{\;2}\vec{r}_{21'}^{\;2}}\right) + (\vec{r}_{12'}^{\;2} + \vec{r}_{21'}^{\;2} - \vec{r}_{1'2'}^{\;2})\ln\left(\frac{\vec{r}_{1'2'}^{\;2}}{\vec{r}_{12}^{\;2}}\right)\right. \end{aligned}$$

$$+(\vec r\,^2_{12}-\vec r\,^2_{11'})\ln\left(\frac{\vec r\,^2_{11'}}{\vec r\,^2_{12'}}\right)+(\vec r\,^2_{12}-\vec r\,^2_{22'})\ln\left(\frac{\vec r\,^2_{22'}}{\vec r\,^2_{21'}}\right)+\vec r\,^2_{21'}\ln\left(\frac{\vec r\,^2_{21'}}{\vec r\,^2_{11'}}\right)+\vec r\,^2_{12'}\ln\left(\frac{\vec r\,^2_{12'}}{\vec r\,^2_{22'}}\right)\Bigg)$$

$$+\frac{1}{2\vec r\,^2_{11'}\vec r\,^2_{22'}}\left(\frac{\vec r\,^2_{12}}{\vec r\,^2_{12'}}\ln\left(\frac{\vec r\,^2_{11'}}{\vec r\,^2_{1'2'}}\right)+\frac{\vec r\,^2_{12}}{\vec r\,^2_{21'}}\ln\left(\frac{\vec r\,^2_{22'}}{\vec r\,^2_{1'2'}}\right)+\frac{\vec r\,^2_{22'}}{\vec r\,^2_{12'}}\ln\left(\frac{\vec r\,^2_{22'}}{\vec r\,^2_{12}}\right)+\frac{\vec r\,^2_{11'}}{\vec r\,^2_{21'}}\ln\left(\frac{\vec r\,^2_{11'}}{\vec r\,^2_{12}}\right)\right),\quad (14)$$

where

$$d=\vec r^{\,2}_{12'}\vec r^{\,2}_{21'}-\vec r^{\,2}_{11'}\vec r^{\,2}_{22'}\;. \quad (15)$$

The gluino part can be obtained from the results of Refs. [2] and [7] by the change of the coefficients: $n_f\to n_M N_c$ for the "non-Abelian" part and $n_f\to -n_M N_c^3$ for the "Abelian" part. Using the subscript M to denote this part, we have

$$g_M(\vec r_1,\vec r_2;\vec\rho)=-g^0_M(\vec r_1,\vec r_2;\vec\rho)=\frac{n_M}{3}\left(\frac{\vec r^{\,2}_{12}}{\vec r^{\,2}_{1\rho}\vec r^{\,2}_{2\rho}}\ln\frac{\vec r^{\,2}_{M}}{\vec r^{\,2}_{12}}+\frac{\vec r^{\,2}_{1\rho}-\vec r^{\,2}_{2\rho}}{\vec r^{\,2}_{1\rho}\vec r^{\,2}_{2\rho}}\ln\frac{\vec r^{\,2}_{1\rho}}{\vec r^{\,2}_{2\rho}}\right)\;, \quad (16)$$

where

$$\ln\vec r^{\,2}_M=-\frac{5}{3}+2\psi(1)-\ln\frac{\mu^2}{4} \quad (17)$$

and

$$g_M(\vec r_1,\vec r_2;\vec r_1{}',\vec r_2{}')=-n_M\frac{1}{\vec r^{\,4}_{1'2'}}\left(\frac{\vec r^{\,2}_{12'}\vec r^{\,2}_{1'2}+\vec r^{\,2}_{11'}\vec r^{\,2}_{22'}-\vec r^{\,2}_{12}\vec r^{\,2}_{1'2'}}{2d}\ln\frac{\vec r^{\,2}_{12'}\vec r^{\,2}_{1'2}}{\vec r^{\,2}_{11'}\vec r^{\,2}_{22'}}-1\right)\;. \quad (18)$$

4 Dipole form of the scalar part

Let us present the scalar contribution to the kernel in the momentum space. Details of the derivation will be given elsewhere [15].

The scalar particles contribute both to the gluon Regge trajectory $\omega(\vec q)$ and to the "real" part $\hat{\mathcal K}_r$ of the kernel:

$$\langle\vec r_1\vec r_2|\hat{\mathcal K}^S|\vec r_1{}'\vec r_2{}'\rangle=\delta(\vec q_1-\vec q_1{}')\delta(\vec q_2-\vec q_2{}')\left(\omega_S(\vec q_1)+\omega_S(\vec q_2)\right)+\langle\vec r_1\vec r_2|\hat{\mathcal K}^S_r|\vec r_1{}'\vec r_2{}'\rangle\;. \quad (19)$$

The contribution to the trajectory can be obtained from the quark contribution through the substitution $n_f\to n_s N_c$ and the change of the fermion polarization operator with the scalar one:

$$\omega_S(\vec q)=\frac{g^4\vec q^{\,2}N_c^2 n_S}{4(2\pi)^{D-1}}\int\frac{d^{(D-2)}k}{\vec k^{\,2}(\vec k-\vec q)^{\,2}}\left[P_s(\vec q)-P_s(\vec k)-P_s(\vec q-\vec k)\right]\;, \quad (20)$$

where

$$P_s(\vec q)=\frac{2\Gamma\left(1-\epsilon\right)\Gamma^2\left(2+\epsilon\right)}{(4\pi)^{2+\epsilon}\epsilon(1+\epsilon)\Gamma\left(4+\epsilon\right)}\vec q^{\,2\epsilon}\;. \quad (21)$$

Here $P_s(\vec{q})$ is the scalar polarization operator where the coupling constant and colour/flavour coefficients have been omitted. It differs from the corresponding fermion operator by the factor $1/(4(1+\epsilon))$.

It is convenient to divide the scalar contribution to the "real" kernel into the "non-Abelian" and "Abelian" parts, $\hat{\mathcal{K}}_r^S = \hat{\mathcal{K}}_n^S + \hat{\mathcal{K}}_a^S$, quite analogously to the quark case [17]. Besides the contribution from production of a couple of scalars, the "non-Abelian" part contains also the scalar loop contribution to one-gluon production. Separately these contributions are rather complicated, but taken together they acquire a simple form:

$$\langle\vec{q}_1\vec{q}_2|\hat{\mathcal{K}}_n^S|\vec{q}_1{}'\vec{q}_2{}'\rangle == \delta(\vec{q}_1+\vec{q}_2-\vec{q}_1{}'-\vec{q}_2{}')\frac{g^4N_c^2n_S}{4(2\pi)^{D-1}}\left[2\left(\frac{\vec{q}_1{}'^{\,2}}{\vec{q}_1^{\,2}\vec{k}^{\,2}}+\frac{\vec{q}_2{}'^{\,2}}{\vec{q}_2^{\,2}\vec{k}^{\,2}}-\frac{\vec{q}^{\,2}}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\right)P_s(\vec{k})\right.$$
$$+\frac{\vec{q}^{\,2}}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\Big(2P_s(\vec{k})+2P_s(\vec{q})-P_s(\vec{q}_1)-P_s(\vec{q}_2)-P_s(\vec{q}_1{}')-P_s(\vec{q}_2{}')\Big)$$
$$\left.+\left(\frac{\vec{q}_1{}'^{\,2}}{\vec{q}_1^{\,2}\vec{k}^{\,2}}-\frac{\vec{q}_2{}'^{\,2}}{\vec{q}_2^{\,2}\vec{k}^{\,2}}\right)(P_s(\vec{q}_1)-P_s(\vec{q}_2)-P_s(\vec{q}_1{}')+P_s(\vec{q}_2{}'))\right]\,. \tag{22}$$

In accordance with the results of Ref. [16], it can be obtained from the "non-Abelian" quark contribution [17] through the same substitutions as in the case of the trajectory. Note that the result (22) is obtained with the prescriptions given in Ref. [14]. As it was explained, we shall apply to it the transformation (1) with the operator $\hat{O}$ given by the formula (10).

In Eqs. (20) and (22) the space-time dimension D is taken different from 4 to regularize the infrared and ultraviolet divergencies. The last ones are removed by the renormalization (8) of the coupling constant both in the leading order trajectory and "real" Born kernel:

$$\omega(\vec{q}) = -\frac{g^2N_c\vec{q}^{\,2}}{2(2\pi)^{D-1}}\int\frac{d^{D-2}k}{\vec{k}^{\,2}(\vec{q}-\vec{k})^2}\,, \tag{23}$$

$$\langle\vec{q}_1,\vec{q}_2|\hat{\mathcal{K}}_r^B|\vec{q}_1{}',\vec{q}_2{}'\rangle = \delta(\vec{q}-\vec{q}\,')\frac{1}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\frac{g^2N_c}{(2\pi)^{D-1}}\left(\frac{\vec{q}_1^{\,2}\vec{q}_2{}'^{\,2}+\vec{q}_2^{\,2}\vec{q}_1{}'^{\,2}}{\vec{k}^{\,2}}-\vec{q}^{\,2}\right)\,. \tag{24}$$

As for the infrared divergency, it is convenient, following Refs. [2] and [8], to regularize it introducing the cut-off λ, tending to zero after taking the limit $\epsilon\to 0$, and picking out in the representations (20) and (23) the domains $\vec{k}^{\,2}\le\lambda^2$ and $(\vec{k}-\vec{q}_i)^2\le\lambda^2$. It is easy to see that the contributions of these domains cancel the contribution of $\hat{\mathcal{K}}_n^S$ from the region $\vec{k}^{\,2}\le\lambda^2$. In the remaining regions we can put $D=4$. Taking into account the transformation (1) with $\hat{O}$ given by the relation (10) and omitting the terms leading to $\delta(\vec{r}_1{}'-\vec{r}_2{}')$, we obtain

$$\langle\vec{q}_1,\vec{q}_2|\hat{\omega}_1+\hat{\omega}_2+\hat{\mathcal{K}}_n^S|\vec{q}_1{}',\vec{q}_2{}'\rangle\to\frac{\alpha_s^2(\mu)N_c^2}{4\pi^3}\frac{n_S}{6}\left[-\delta(\vec{q}_1-\vec{q}_1{}')\delta(\vec{q}_2-\vec{q}_2{}')\int d\vec{k}\,\Big(2V_S(\vec{k})\right.$$

$$+V_S(\vec{k},\vec{k}-\vec{q}_1)+V_S(\vec{k},\vec{k}+\vec{q}_2)\Big)+2\delta(\vec{q}-\vec{q}\,')\left(V_S(\vec{k})+V_S(\vec{k},\vec{q}_1)+V_S(\vec{k},\vec{q}_2)\right)\Bigg]\ , \tag{25}$$

where

$$V_S(\vec{k})=\frac{1}{2\vec{k}^{\,2}}\left(\ln\left(\frac{\vec{k}^{\,2}}{\mu^2}\right)-\frac{8}{3}\right)\ , \tag{26}$$

$$V_S(\vec{k},\vec{q})=-\frac{\vec{k}\vec{q}}{2\vec{k}^{\,2}\vec{q}^{\,2}}\left(\ln\left(\frac{\vec{k}^{\,2}\vec{q}^{\,2}}{\mu^2(\vec{k}-\vec{q})^2}\right)-\frac{8}{3}\right)+\frac{1}{4\vec{k}^{\,2}}\ln\left(\frac{\vec{q}^{\,2}}{(\vec{k}-\vec{q})^2}\right)+\frac{1}{4\vec{q}^{\,2}}\ln\left(\frac{\vec{k}^{\,2}}{(\vec{k}-\vec{q})^2}\right)\ . \tag{27}$$

The "Abelian" part can be written in the form

$$\langle\vec{q}_1,\vec{q}_2|\hat{\mathcal{K}}_a^S|\vec{q}_1\,',\vec{q}_2\,'\rangle=\delta(\vec{q}-\vec{q}\,')\frac{g^4N_c^2n_S}{4(2\pi)^{D-1}}\frac{1}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\int_0^1dx\int\frac{d^2k_1}{(2\pi)^{D-1}}x(1-x)$$
$$\times\left(\frac{\vec{q}_1^{\,2}-2(\vec{q}_1\vec{k}_1)}{\sigma_{11}}+\frac{\vec{q}_1^{\,2}-2(\vec{q}_1\vec{k}_2)}{\sigma_{21}}\right)\left(\frac{2(\vec{q}_2\vec{k}_1)+\vec{q}_2^{\,2}}{\sigma_{12}}+\frac{2(\vec{q}_2\vec{k}_2)+\vec{q}_2^{\,2}}{\sigma_{22}}\right)\ , \tag{28}$$

where $\vec{k}_1+\vec{k}_2=\vec{k}=\vec{q}_1-\vec{q}_1\,'=\vec{q}_2\,'-\vec{q}_2$,

$$\sigma_{11}=(\vec{k}_1-x\vec{q}_1)^2+x(1-x)\vec{q}_1^{\,2},\quad\sigma_{21}=(\vec{k}_2-(1-x)\vec{q}_1)^2+x(1-x)\vec{q}_1^{\,2}\ ,$$
$$\sigma_{12}=(\vec{k}_1+x\vec{q}_2)^2+x(1-x)\vec{q}_2^{\,2},\quad\sigma_{22}=(\vec{k}_2+(1-x)\vec{q}_2)^2+x(1-x)\vec{q}_2^{\,2}\ . \tag{29}$$

This part agrees with the forward QED kernel considered in Ref. [16]. It contains neither ultraviolet nor infrared singularities and therefore it does not require neither regularization nor renormalization. Therefore we can use from the beginning the physical space-time dimension $D=4$ and the renormalized coupling constant α_s. Restricting ourselves to the dipole form, we can perform the change

$$\langle\vec{q}_1,\vec{q}_2|\hat{\mathcal{K}}_a^S|\vec{q}_1\,',\vec{q}_2\,'\rangle\to\delta(\vec{q}-\vec{q}\,')\frac{{\alpha_s}^2(\mu)N_c^2n_s}{4\pi^3}\frac{1}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\int_0^1dx\int\frac{d^2k_1}{(2\pi)}\frac{x(1-x)}{\sigma_{11}\sigma_{22}}(\vec{q}_1^{\,2}-2(\vec{q}_1\vec{k}_1)(\vec{q}_2\vec{k}_2)+\vec{q}_2^{\,2})\ . \tag{30}$$

The transformation of Eqs. (25) and (30) into the dipole form can be easily done with the help of the integrals presented in Ref. [8]. As a result we have

$$g_S(\vec{r}_1,\vec{r}_2;\vec{\rho})=-g_S^0(\vec{r}_1,\vec{r}_2;\vec{\rho})=\frac{n_S}{12}\left(\frac{\vec{r}_{12}^{\,2}}{\vec{r}_{1\rho}^{\,2}\vec{r}_{2\rho}^{\,2}}\ln\frac{\vec{r}_S^{\,2}}{\vec{r}_{12}^{\,2}}+\frac{\vec{r}_{1\rho}^{\,2}-\vec{r}_{2\rho}^{\,2}}{\vec{r}_{1\rho}^{\,2}\vec{r}_{2\rho}^{\,2}}\ln\frac{\vec{r}_{1\rho}^{\,2}}{\vec{r}_{2\rho}^{\,2}}\right)\ , \tag{31}$$

where

$$\ln r_S^2=-\frac{8}{3}+2\psi(1)-\ln\frac{\mu^2}{4}\ , \tag{32}$$

and

$$g_S(\vec{r}_1,\vec{r}_2;\vec{r}_1{}',\vec{r}_2{}') = \frac{n_S}{2}\frac{1}{\vec{r}^{\,4}_{1'2'}}\left(\frac{\vec{r}^{\,2}_{12'}\vec{r}^{\,2}_{1'2}}{d}\ln\frac{\vec{r}^{\,2}_{12'}\vec{r}^{\,2}_{1'2}}{\vec{r}^{\,2}_{11'}\vec{r}^{\,2}_{22'}} - 1\right) , \tag{33}$$

with d defined in Eq. (15). At that $g_S(\vec{r}_1,\vec{r}_2;\vec{\rho})$ is determined by the "non-Abelian" part (25) and $g(\vec{r}_1,\vec{r}_2;\vec{r}_1{}',\vec{r}_2{}')$ by the "Abelian" part (30). It is evidently conformal invariant.

5 Discussion

The results (11)-(18) and (32)-(33) permit to write the dipole form of the BFKL kernel in sypersymmetric Yang-Mills theories with any N. The most interesting is the $N = 4$ case, with $n_M = 4, \quad n_S = 6$, where one can hope on conformal invariance. Unfortunately, the results presented in this paper do not show this property. There are terms violating conformal invariance both in $g(\vec{r}_1,\vec{r}_2;\vec{\rho})$ and in $g(\vec{r}_1,\vec{r}_2;\vec{r}_1{}',\vec{r}_2{}')$. Let us remind, however, that the NLO kernel is not unambiguously defined. The transformation (1), accompanied by the corresponding transformation of the impact factors, does not change scattering amplitudes. The meaning of these transformations is the redistribution of the radiative corrections between the kernel and the impact factors. In this aspect the situation is quite analogous to the possibility to redistribute the radiative corrections between the anomalous dimension and the coefficient functions. The transformation (1) can be used for the elimination of the terms violating conformal invariance. Till now the possibility of a complete elimination of these terms is neither proved nor disproved.

Recently the paper [18] appeared in the web. In this paper the NLO gluon contribution to the BK kernel is found. The result of Ref. [18] is not in accord with that of Ref. [8]. One could think that the disagreement can be removed using the transformation (10), but the result of Ref. [18] does not agree also with that of Ref. [19] confirmed in Ref. [20] and Ref. [21]. We hope to turn to this discrepancy elsewhere.

Acknowledgment

V.S.F. thanks the Dipartimento di Fisica dell'Università della Calabria and the Istituto Nazionale di Fisica Nucleare, Gruppo Collegato di Cosenza, for the warm hospitality while part of this work was done and for the financial support.

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